We remind the teams that the answer to each problem is an integer number (from 0000 to 9999), which should be written on the answer sheet.

If your answer is rational, but not an integer, the answer is the sum of the numerator and denominator of the simplest form of the fraction.

In the first 15 minutes any problem can be marked as a Jolly problem (for double points), for which the team hasn't provided a good solution yet.

1. When Mr X ended up on the deserted island, the number of hairs on his head was 200 000, and each of them was $5\,cm$ long. His hair grew $0.5\,mm$ per day, but Mr X didn't cut his hair, because on the one hand he didn't have anything to cut it with, and on the other hand he lost 50 hairs each day. How many days later reached its maximum the total length of Mr X's hair? **(20 points)**

2. The sum of the squares of 100 consecutive positive integers is the same as the sum of the squares of the next 99 positive integers. Find the biggest of these 199 positive integers. **(25 points)**

3. The angles of the almost equilateral triangle T are $59.99°$, $60°$ and $60.01°$. Let T_1 be the triangle defined by the feet of the altitudes of triangle T. Let T_2 be the triangle defined by the feet of the altitudes of triangle T_2, and so on. Find the smallest n such that T_n has an obtuse angle. **(25 points)**

4. The lengths of the legs of a right angled triangle are consecutive integers, and the length of the hypotenuse of the triangle is an integer greater than 5. What is the smallest possible perimeter of the triangle? **(25 points)**

5. In a factory it was noted that the performance of the workers per hour is proportional to the square of the number of hours they have slept that day. How many hours should the workers sleep every day, so that the output of the factory will be maximal? (The workers either work or sleep at each moment of a day.) **(25 points)**

6. There are two dice with detachable numbers from 1 to 6. We take down the twelve numbers, put them in a hat, and then we glue them back on the two cubes one by one randomly (on each face we glue one number). Now we roll the two dice and take the sum of the results. What is the chance the we get 7? **(30 points)**

7. Let A denote the volume of the regular tetrahedron with unit edges. Let B denote the volume of the regular octahedron with unit edges. Find ratio A/B. The answer is the sum of the numerator and denominator in the simplest form of the fraction. **(30 points)**

8. p, q and r are primes satisfying $5 \le p < q < r$. Find the product of the primes knowing that $2p^2 - r^2 \ge 49$ and $2q^2 - r^2 \le 193$. **(30 points)**

9. Determine the sum of the squares of the irrational numbers x for which $x^2 + 2x$ and $x^3 - 6x$ are both rational numbers. **(30 points)**

10. Let x_1 and x_2 denote the roots of polynomial $3x^2 + ax + b$. Let y_1 and y_2 denote the values of function $f(x) = \frac{5x+2}{2x-1}$ at x_1 and x_2. Find the value of $a^2 + b^2$ knowing that y_1 and y_2 satisfy the equation $7y^2 - 5y - 11 = 0$. **(35 points)**

11. It is known that for $n \geq k$ polynomial $P_k(x) = (x^k - 1) \cdot (x^{k-1} - 1) \cdot \ldots \cdot (x - 1)$ divides polynomial $P_{n,k}(x) = (x^n - 1)(x^{n-1} - 1) \cdot \ldots \cdot (x^{n-k+1} - 1)$. $Q_{n,k}(x)$ denotes the quotient polynomial. Find the value of $Q_{10,5}(1)$. **(35 points)**

12. Take 1000 lines in the plane such that no three are concurrent and no two are parallel. These lines divide the plane into regions. What is smallest possible number of the triangles we get among the regions? **(35 points)**

13. Integer numbers $x_0, x_1, \ldots, x_{5000}$ satisfy equations $x_i^2 = 1 + x_{i-2}x_{i-1}$ ($i = 0, 1, \ldots, 5000$, indices are meant modulo 5001). What is the greatest possible value of $x_0^2 + x_1^2 + \cdots + x_{5000}^2$?
(40 points)

14. In cyclic octagon $ABCDEFGH$ sides $AB = BC = CD = DE = 1$ and $EF = FG = GH = HA = 3$. Take a square such that its incircle is the same as the circumcircle of the octagon. Find the difference between the area of the square and the octagon. **(40 points)**

15. In tetrahedron $ABCD$ faces ABC and ABD are orthogonal to each other, and the same holds for faces ACD and BCD. The area of triangles ABC and ABD are 10 and 11 units respectively. The areas of the four faces are four distinct integers. Find the surface area of the tetrahedron.
(40 points)

16. Let S be a square shaped region, and $n \geq 4$ integer. A point X inside the square is called n-ray partitional, if there exist n rays starting from X such that they divide the square into n triangles of equal area. How many points inside the square are 100-ray partitional, but not 60-ray partitional? **(45 points)**

17. Define sequence a_n the following way: $a_1 = \sqrt{2}$, $a_{n+1} = \frac{\sqrt{3}a_n - 1}{a_n + \sqrt{3}}$. Find the tenth power of a_{10}. The answer to this question is the sum of the numerator and denominator of the simplest form of the resulting fraction. **(45 points)**

18. Let α denote the greatest root of equation $x^3 - 3x^2 + 1 = 0$. What is the modulo 17 remainder of $\lfloor \alpha^{2014} \rfloor$?
(45 points)

19. Suppose that the edges of a graph on 10 vertices can be colored using two colors such that the graph does not contain a triangle with the same color for all three edges. What is the maximal number of edges the graph can have? **(50 points)**

20. Inside unit square $ABCD$ we choose two points P and Q. Determine the smallest possible value of the following expression:

$$[13(PA + QC) + 14PQ + 15(PB + QD)]^2$$

(50 points)